Numerical Study of Laminar Natural Convection of Air in a Cavity allowing for Variable Fluid Properties

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Numerical simulations are presented for laminar buoyancy-driven flows and heat transfer in a cavity using the penalty finite element method. The simulations were accomplished for Rayleigh numbers between $10^3$ and $10^6$ and Prandtl number of 0.706. For most of the work, the usual Boussinesq approximation has been used. However, in order to demonstrate the effects of temperature-dependent physical properties for natural convection in a cavity, a modified Boussinesq approximation with temperature-dependent viscosity and thermal conductivity has been used.

The formulation was based on primitive variables. The non-linear equations were solved using the Picard type of iterative method. The iterative procedure was repeated until the flow field computed in two consecutive iterations differed by less than a specified tolerance. It is necessary to start with a low value of Rayleigh number: once convergence has been obtained for the given value of Rayleigh number, its value is incremented by a factor of 10. In addition, the integrals were evaluated numerically with the aid of Gaussian quadrature and the resulting set of non-linear equations was linearised using the Newton iteration technique. Quadratic rectangular elements with nine nodes were employed and a non-uniform mesh structure with a finer mesh near the boundaries was used.

Simulations for the cavity with Boussinesq approximation are presented here primarily for the purpose of validating the calculations with experimental results reported in the literature. Numerical results are presented in terms of velocity components, temperature profiles, stream functions and Nusselt numbers. The results are compared with recent publications and good agreement has been found.

There is little significant difference in isotherms and streamlines with variable properties. The most significant feature is the lower (magnitude) temperature gradient near the vertical surfaces with the variable thermal conductivity. In order to show the variation of the primitive variables ($U$, $V$ and $\theta$), they are plotted for three different regions of the square cavity ($X=0.23$ in the hot region, $X=0.5$ in the middle and $X=0.72$ in the cold region). Results illustrate the variation of velocity component $U$ at $X=0.23$, $0.5$, $0.72$ with $Y$ for various values of the Rayleigh numbers. The results show that near the hot wall the magnitude of the horizontal velocity component tends to be highest for constant properties, particularly at low and moderate Rayleigh numbers. Similarly, near the cold wall, the constant properties fluid tends to exhibit the lowest value of $U$.

The plots of velocity component $V$ at $Y=0.23$, $0.5$, $0.72$ ($Y=0.23$ is in the lower region, $Y=0.5$ in the middle region and $Y=0.72$ in the upper region.), at all three levels it is found that, for the case of constant properties, the magnitude of $V$ is higher near the hot surface and lower near the cold surface than for the variable property cases.

The variation of dimensionless temperature $\theta$ at $Y=0.23$, $0.5$, $0.72$ is plotted versus $X$ for different Rayleigh numbers. As shown in these figures, the dimensionless temperature variations are affected by both the thermal conductivity and the viscosity. However, the thermal conductivity has a greater effect than the viscosity, and is in the opposite sense.

The variations of local Nusselt number on the hot ($X=0$) and the cold ($X=1$) surfaces are affected very significantly by the varying thermal conductivity but insignificantly by the viscosity.